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Crossover from the ballistic to the resonant tunneling transport for an ideal one-dimensional quantum ring with spin-orbit interaction

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Quantum interference effects in nanometric devices provide suitable means for controlling spin at mesoscopic scales. As discussed by Foldi *et al.* [Phys. Rev. B **71**, 033309 (2005)], in a quantum ring connected with two external leads, the spin properties of an incoming electron are modified by the spin-orbit interaction (SOI), resulting in a transformation of the qubit state carried by the spin. The ring acts as a one-qubit spintronic quantum gate whose properties can be varied by tuning the Rashba parameter of the SOI, as well as by changing the size of the ring. However continuous transport experiment cannot be utilized as quantum information processing because in spintronics one has to handle a single electron which carries the information in its spin. Thus, starting from the ballistic structure by Foldi *et al.*, we propose a device which works in the regime of resonant tunneling and is able to handle a single electron. We discuss the crossover between the two different regimes.

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I. INTRODUCTION

In the last decade enormous attention has been devoted toward control and engineering of spin degree of freedom in nanostructures, usually referred to as spintronics.[1,2] In fact physicists are trying to exploit the "spin" of the electron rather than its charge, in order to create a remarkable new generation of "spintronic" devices[1,3] that are able to control the electron spin in submicrometric devices.

Since the pioneering work of Datta and Das,[4] in which they proposed a spin transistor based on the spin-orbit interaction (SOI), there has been a great deal of theoretical and experimental work concerning the possibility of controlling the electron spin by means of an electric field with or without the help of magnetic field and magnetic material.[5–7]

A. Rashba Spin-orbit Interaction

Usually, the electric field does not act on the spin. But if a device is prepared in a semiconductor heterostrucrure with an asymmetrical-interface electric field (Fig. 1, right), Rashba spin-orbit interaction (RSOI) will occur.[8,9] The RSOI, a relativistic effect at the low-speed limit, is essentially the influence of an external field on a moving spin and can be seen as the interaction of the electron spin with the magnetic field $B_{\rm eff}$ appearing in the rest frame of the electron. It can couple the spin degree of freedom to its orbital motion, thus making it possible to control the electron spin in terms of external electric fields or gate voltages. In the case of quantum heterostructures, a two-dimensional electron gas [(2DEG) in the x,y plane] is entrapped in a semiconductor quantum well due to the band offsets at the interface of two different materials. Thus the SOI originated intrinsically will be due to an effective electric field E along $\hat{z}$ and the SOI Hamiltonian will take the form[10,11]

$$\hat{H}_{\rm SO}^{\alpha}=\frac{\alpha}{\hbar}[\sigma_{\bf x}{\bf p_y}-\sigma_{\bf y}{\bf p_x}], \quad (1)$$

where σ_x and σ_y are the Pauli matrices. The parameter α represents the average electric field along the z direction. For an InGaAs-based system α can be controlled by a gate voltage with values in the range $(0.5-2.0)\times 10^{-11}$ eV m (see Ref. 12), while the highest value of α in 2DEGs is close to 10^{-10} eV.[13,14]

B. Spintronics Logic Gates

In order to implement quantum operations on single electron spins, appropriate elementary logical operations (*gates*) are necessary[15] that operate on this type of qubits. Recently[16] it was shown that a quasi-one-dimensional (1D) ring connected with two external leads (Fig. 1, top) made of a semiconductor structure in which RSOI is the dominant spin-flipping mechanism can render such a gate. The authors of Ref. 16 showed that the ring, in ballistic regime, acts as a one-qubit spintronic quantum gate, i.e., a large class of unitary transformations can be attained with already one ring, or a few rings in series, including the important cases of the Z, X, and Hadamard gates. By choosing appropriate parameters the spin transformations can be made unitary, which corre-

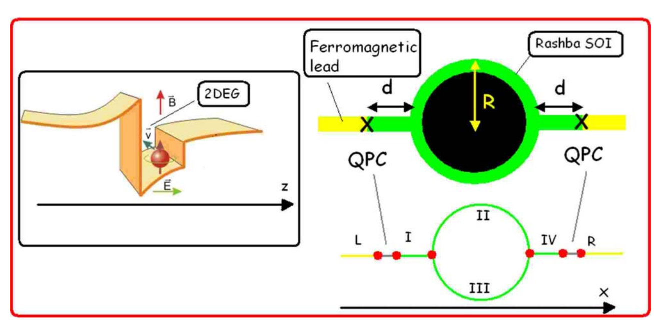

FIG. 1. (Color online) (Left) The Rashba effect originates from the macroscopic electric field in a semiconductor quantum well; in the inset a typical conduction-band profile of a semiconductor quantum well is depicted. (Right) Schematic of the quantum ring with two legs interrupted by two quantum point contacts. The device proposed here is made of quasi-1D wires of width W ranging between ∼25 nm and 50 nm and a ring radius ∼200 nm.

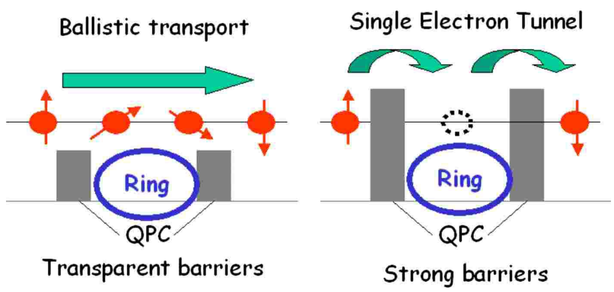

FIG. 2. (Color online) Schematic model for the Ballistic transport and for the single electron tunneling.

sponds to lossless gates. However in the discussed structures,[16,17] it is very difficult to handle a single electron while usually continuous transport experiment cannot be utilized as quantum information processing. Thus one has to handle a single electron for quantum information processing because information is carried by the spin of a single electron.

C. Summary

In this paper we propose a device where two quantum point contacts (QPCs) (see Fig. 1) are placed in each lead. Modulating the strength of the barriers corresponding to the QPCs, we have a crossover from the ballistic transport (transparent barriers) to a resonant tunneling regime. The latter regime is characterized by the fact that just a single electron tunnels time by time through the ring that can be now viewed as a lateral quantum dot (Fig. 2).

In this paper we first analyze the model which describes the suggested device by calculating the relevant parameters in the physics of ideal 1D ballistic devices (the legs and the ring). Next we use the Landauer formalism with the aim of calculating the conductance, once the transmission amplitudes obtained by applying the quantum waveguide approach are known. Thus we will be able to discuss the effects of the QPCs on the conductance and the consequences in the spin polarization of the transmitted electrons. Some interesting effects due to the multiple reflections and to the interference will be presented as results, which we discuss from a spintronics point of view.

II. THEORETICAL MODEL FOR THE LEGS

The arms (I and IV) and the leads (L and R or source and drain, respectively) will be assumed as ballistic one-dimensional wires where electrons propagate freely down a clean narrow pipe and electronic transport with no scattering can occur. Because the Rashba electric field can be reasonably assumed to be uniform and directed along the z axis, the relevant effects of the RSOI can be obtained by the simplified Hamiltonian[18,19]

$$\hat{H}_c = (\alpha \hat{\sigma}_y p_x)/\hbar = (\hbar k_R p_x \hat{\sigma}_y)/m^*,$$

where $k_R \equiv (m^* \alpha)/\hbar^2$.

It follows that the Rashba subbands splitting in the energies, in the first-order approximation, reads as

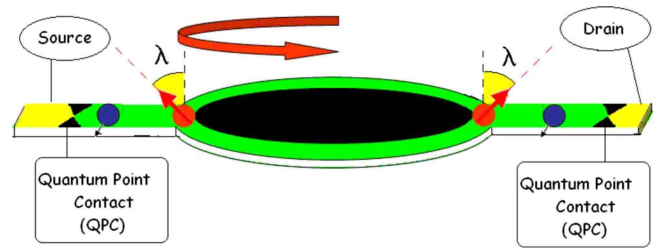

FIG. 3. (Color online) The ring and the spin polarization of the eigenstates in the ring.

$$\varepsilon_{k,s_y} = \hbar \frac{\hbar^2}{2m^*}[(k \pm k_R)^2 - k_R^2]. \tag{2}$$

Here $s_y = +1$ (-1) correspond to $\chi_y^{\uparrow}$ $(\chi_y^{\downarrow})$ spin eigenfunctions along the y direction. Hence we can conclude that four-split channels are present for a fixed Fermi energy ε_F, corresponding to $\pm p_x$ and $s_y = \pm 1$. Thus the wave functions in the wires are built starting from

$$\phi_{q,\uparrow} = e^{iqx} e^{ik_R x} \chi_{\uparrow}^{y}, \quad \phi_{q,\downarrow} = e^{iqx} e^{-ik_R x} \chi_{\downarrow}^{y},$$

where $\hbar^2 k^2 = (2m^* \varepsilon_F)$ and $q = \sqrt{k_R^2 + k^2}$.

III. THEORETICAL MODEL FOR THE RING

In the cylindrical coordinate, more suitable to the aim of studying the ring region, the RSOI Hamiltonian in Eq. (1) becomes

$$H_\alpha = -\frac{\alpha}{r}\sigma_r\left(i\frac{\partial}{\partial\varphi}\right) + i\alpha\sigma_\varphi\frac{\partial}{\partial r} - \frac{i}{2}\frac{\alpha}{r}\sigma_\varphi. \tag{3}$$

Here $\sigma_r = \cos\varphi\sigma_x + \sin\varphi\sigma_y$ and $\sigma_\varphi = -\sin\varphi\sigma_x + \cos\varphi\sigma_y$. In the case of a thin ring we can neglect the second term in the right-hand side[16,20] of Eq. (3) and assume $r = R$, in agreement with the result in Eq. (2) of Ref. 20. In the presence of SOI, the Hamiltonian operator for a one-dimensional ring reads[21] as

$$H = -\hbar\omega_R(\partial^2/\partial\varphi^2) - i\hbar\omega_\alpha\sigma_r(\partial/\partial\varphi) - i\hbar\omega_\alpha\sigma_\varphi/2, \tag{4}$$

where the parameter $\omega_R \equiv \hbar/(2m^* R^2)$ and $\omega_\alpha = \alpha/(\hbar R)$ is the frequency associated with the RSOI. We now introduce the preserved spin

$$\sigma_\lambda = \sigma_z \cos(\lambda) - \sigma_r \sin(\lambda),$$

where the angle λ is defined as (Fig. 3) $\cot(\lambda) = \omega_R/\omega_\alpha$. The energy spectrum ε is given by

$$\tilde{\varepsilon}_{\mu,s_\lambda} = \hbar\omega_R\left(\mu + \frac{\Phi_{AC}^{\pm}}{2\pi}\right)^2,$$

where $\tilde{\varepsilon} \equiv \varepsilon_F - \hbar\omega_\alpha^2/\omega_R$ and $\Phi_{AC}^{\pm} = -\pi(1 \pm \omega_T/\omega_R)$ are the Aharonov-Casher phases which are acquired, as the two spin states evolve in the ring in the presence of the Rashba electric field.[22,23] We set $\omega_T^2 = \omega_R^2 + \omega_\alpha^2$ while the unnormalized eigenstates are

$$\psi_{\mu,+} = e^{i\mu\varphi}\left[\cos\left(\frac{\theta}{2}\right)\chi_z^{\uparrow} + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)\chi_z^{\downarrow}\right],$$

$$\psi_{\mu,-} = e^{i\mu\varphi}\left[\sin\left(\frac{\theta}{2}\right)\chi_z^{\uparrow} + e^{i\varphi}\cos\left(\frac{\theta}{2}\right)\chi_z^{\downarrow}\right],$$

with $\tan(\theta/2)=(\omega_R-\omega_T)/\omega_\alpha$. It follows that for a fixed value of the Fermi energy ε_F, there are four different eigenstates $\psi^{s,\gamma}_{\pm\mu}$, i.e., particles can go through the ring with four different wave numbers, depending on spin (s) and direction of motion ($\pm$).

IV. THEORETICAL APPROACH: FROM THE TRANSMISSION TO THE CONDUCTANCE

As discussed in Ref. 22, in the Landauer framework,[24] the quantum-mechanical transmission amplitude is related to two-probe ballistic conductance. The spin-dependent conductance through the ring may be expressed in terms of the transmission probability T, as $G=(e^2/h)T$. In order to calculate the transmission probability through the leg-ring-leg (–*O*–) device, we use the local coordinate system for each circuit. After evaluating the phase difference, we approach the scattering problem with the QPCs, using the quantum waveguide theory.[25] The QPCs are modeled by introducing a Dirac delta potential in the Hamiltonian with a strength U at a distance d from the leg-ring intersections. Thus we write the Hamiltonian in the whole device as

$$\hat{H}_{\rm I} = \frac{p_x^2}{2m^*} + \hat{H}^{\alpha}_{\rm SO} + U\delta(x+d) \ \text{left} \ x<0,$$

$$\hat{H}_{\rm II} = -\hbar\omega_R(\partial^2/\partial\varphi^2) + \hat{H}^{\alpha}_{\rm SO} \ \text{ring} \ \pi>\varphi>0,$$

$$\hat{H}_{\rm III} = -\hbar\omega_R(\partial^2/\partial\varphi^2) + \hat{H}^{\alpha}_{\rm SO} \ \text{ring} \ 2\pi>\varphi>\pi,$$

$$\hat{H}_{\rm IV} = \frac{p_x^2}{2m^*} + \hat{H}^{\alpha}_{\rm SO} + U\delta(x-d) \ \text{right} \ x>0. \qquad (5)$$

The corresponding wave function, e.g., for a spin up injected in the left lead, has to be written as

$$\Psi_L = \phi_{q,\uparrow} + \sum_{s'} r^{\uparrow,s'}\phi_{-q,s} \ \text{left} \ x<-d,$$

$$\Psi_{\rm I} = \sum_{s'} A_{\rm I}^{\uparrow,s}\phi_{q,s} + \sum_{s'} A_{\rm I}^{\uparrow,s}\phi_{-q,s} \ \text{left} \ -d<x<0,$$

$$\Psi_{\rm II} = \sum_{s'} A_{\rm II}^{\uparrow,s}\psi_{\mu,s} + \sum_{s'} A_{\rm II}^{\uparrow,s}\psi_{-\mu,s} \ \text{ring} \ \pi>\varphi>0,$$

$$\Psi_{\rm III} = \sum_{s'} A_{\rm III}^{\uparrow,s}\psi_{\mu,s} + \sum_{s'} A_{\rm III}^{\uparrow,s}\psi_{-\mu,s} \ \text{ring} \ 2\pi>\varphi>\pi,$$

$$\Psi_{\rm IV} = \sum_{s'} A_{\rm IV}^{\uparrow,s}\phi_{q,s} + \sum_{s'} A_{\rm IV}^{\uparrow,s}\phi_{-q,s} \ \text{right} \ d>x>0,$$

$$\Psi_R = \sum_{s'} t^{\uparrow,s'}\phi_{q,s} \ \text{left} \ x>d. \qquad (6)$$

Here we write two different expressions of Ψ at the left and at the right sides of the barriers because, as it is known,[26] also the δ-potential well splits the space into two parts.

The Griffith[27] boundary condition states that the wave function is continuous and that the current density is conserved at each intersection. The resulting set of linear equations leads to a relation between the expansion coefficients $A_p^{s,s'}$ in the different domains (p labels the region I, … ,IV and we use $t^{s,s'}$ and $r^{s,s'}$ for the drain and source, respectively). From these $t^{s,s'}$ we obtain the transmission coefficients $T^{s,s'}$, where s is the spin of the injected electron in the source and s' is the spin of the exiting one in the drain.

V. SOURCE AND DRAIN WAVE FUNCTIONS

In the ballistic regime the –*O*– nanodevice proposed here has to allow a significant class of spin transformations to be described now in the fixed S_y basis.[16] We focus here on the transmission properties of the ring, when an electron from the source is injected in the left probe

$$\psi_S = e^{ikx}(f^{\uparrow}\chi_y^{\uparrow}e^{ik_Rx} + f^{\downarrow}\chi_y^{\downarrow}e^{-ik_Rx}).$$

The emerging wave function in the drain can be written as

$$\psi_D = e^{ikx}(d^{\uparrow}\chi_y^{\uparrow}e^{ik_Rx} + d^{\downarrow}\chi_y^{\downarrow}e^{-ik_Rx}),$$

where $\mathbf{d}=\mathbf{T}\cdot\mathbf{f}$.

VI. TRANSPARENT BARRIERS

The results for $U=0$ are the same ones reported in Ref. 22 [Eqs. (6) and (7)] and can be expressed in a simple analytical form. In this case the transmission matrix is given by[16,17] $\mathbf{T}=Te^{-i\delta}\mathbf{U}$, where $\mathbf{U}$ is a unitary unimodular matrix. This unitary part,

$$\mathbf{U} = \begin{pmatrix} \cos(\gamma_{\rm ring}) & -\sin(\gamma_{\rm ring}) \\ \sin(\gamma_{\rm ring}) & \cos(\gamma_{\rm ring}) \end{pmatrix}, \quad \gamma_{\rm ring} = \Phi_{\rm AC}, \qquad (7)$$

performs a nontrivial spin transformation; it is independent of the wave vector and rotates the spin around x by an angle $2\gamma_{\rm ring}$. By changing the strength of the RSOI, according to Ref. 16, the values of $2\gamma_{\rm ring}$ can be varied. Moreover, as we show on top left of Fig. 4 (black line), periodic oscillations in the conductance versus the Fermi energy are present.

If we assume the injected spin polarized along y [$f^{\downarrow}=0$ and $\mathbf{f}\equiv(1;0)$] the emerging spin is given by $\mathbf{d}=(t^{\uparrow\uparrow}f^{\uparrow};t^{\uparrow\downarrow}f^{\uparrow})$. Thus we obtain

$$\cos(\gamma) \equiv \langle S_y\rangle = \frac{|t^{\uparrow,\uparrow}|^2 - |t^{\uparrow,\downarrow}|^2}{|t^{\uparrow,\uparrow}|^2 + |t^{\uparrow,\downarrow}|^2} = \cos(2\gamma_{\rm ring}).$$

VII. NONTRANSPARENT BARRIERS

When we introduce the barriers (e.g., acting on the voltage of the QPCs gate) the transmission versus ε_F, according the SET regime, has some peaks for fixed values of the Fermi energy, while the transmission is strongly suppressed elsewhere (Fig. 4, top; left red line). Moreover we cannot write the transmission matrix in the simple form $T\mathbf{U}$ while the spin rotation, induced by the AC phase acquired in the

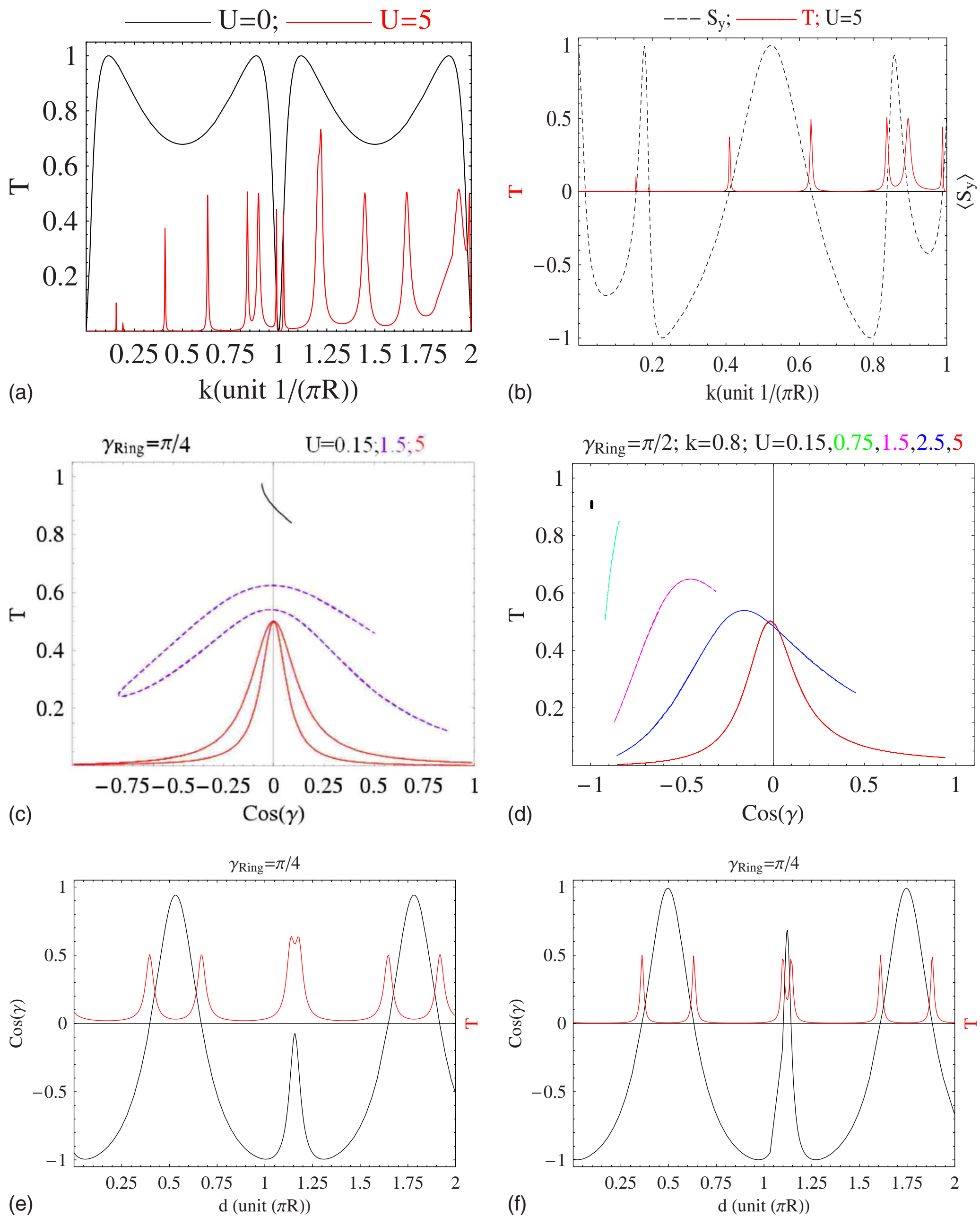

FIG. 4. (Color online) In the transmission (top left) versus k the typical interferometric oscillations are shown for strong-coupling (SC) regime (black line) while the presence of barriers (QPCs) (red line) induces resonant peaks. (Top right) We show the spin polarization along the y direction (dashed line) and compare it with the conductance oscillations in the case of strong barriers. In the middle panels we show how the crossover from strong to weak coupling induces a strong correlation between the peaks and the spin polarization $[\cos(\gamma)\equiv\langle S_y\rangle]$, so that the spin polarization of the emerging electrons does not depend any more on the spin polarization of the injected electron and on the AC phase. This is the weak-coupling limit where the spin of the transmitted electron does not depend on the polarization of the injected current. On the contrary, in the SC regime in the quantum ring connected with two normal external leads, the spin properties of the incoming electrons are modified by the SOI via the AC phase, resulting in a transformation of the qubit state carried by the spin. If the barriers are perfectly transparent we reproduce the known results of Ref. 16, with a rotation angle related to the AC phase acquired by the electron traveling in the quantum ring; while in certain cases we find, however, that T is unitary. Here we assumed $\gamma_{\rm ring}=\pi/2$ in all the panels, while in the bottom panels the value of k is expressed in terms of the ring radius as $k\sim 0.8/(\pi R)$. (Middle) The transmission as a function of the spin polarization is reported, in the left panel, for the strong-coupling regime (black line), where we have $\cos(\gamma)=0$; while in the right panel, for the strong-coupling regime (black line), we have $\cos(\gamma)\sim -1$. In the weak-coupling limit ($U=5$) the transmission has a peak at $\cos(\gamma)=0$, with a center and a shape which do not depend on the strong-coupling polarization. (Bottom panel) The transmission (red, lighter line) versus the ring-QPC distance d. The typical interferometric oscillations are shown for an intermediate coupling regime ($U=3$, left) and for a strong-coupling regime ($U=5$, right). We also show the spin polarization along the y direction (black line) and compare it with the conductance oscillations proportional to T.

ring, is modified because the transmission is usually strongly suppressed. As we show on the top right of Fig. 4, many peaks in the transmission are located where the spin polarization $\langle S_y \rangle$ vanishes. Thus, when the strength of the barriers U increases, the transmission peaks are shifted toward $\cos(\gamma)=0$ (Fig. 4, middle panels). We can deduce that this loss of information is due to the multiple reflection of the electrons inside the device which behaves as a lateral quantum dot. Moreover also the interferometric path is largely modified.

Starting from a different point of view we can assume that in the weak-coupling limit, the spin transmission is dominated by the spin properties of the bound states in the isolated dot between the two QPCs, which are now assumed as strong tunnel barriers. The role played by the distance d between QPCs and the ring along the two leads is however relevant because of the RSOI action which is also in this region. In the bottom of Fig. 4 we show the d dependent oscillations in the transmission and spin polarization.

We now focus on the intermediate (crossover) regime, where the transport involves a single electron, while the spin polarization of the exiting electron is correlated with the spin polarization of the injected one. In this case some transmission peaks correspond to spin polarization of the transmitted electron with γ between 0 and $\pi/2$. In Fig. 5 we report the position of the peaks in the plane $\gamma_{\rm ring}-\gamma$ by showing that the rotation angle γ can be modulated between 0 and $\pi/2$, as in the case of perfectly transparent barriers, by choosing the significant peaks in the conductance. Moreover, for fixed values of the parameters and $\gamma_{\rm ring}$, we can find some peaks with different rotation angles mainly located near $\gamma=\pi/2$.

Thus we can conclude that the ballistic *spintronic single-qubit gate based on a quantum ring with SOI* is able to operate also in the single electron-tunneling regime; but it needs a specific setting of the parameters and an accurate choice of the conductance peak, in order to obtain the wished rotation angle.

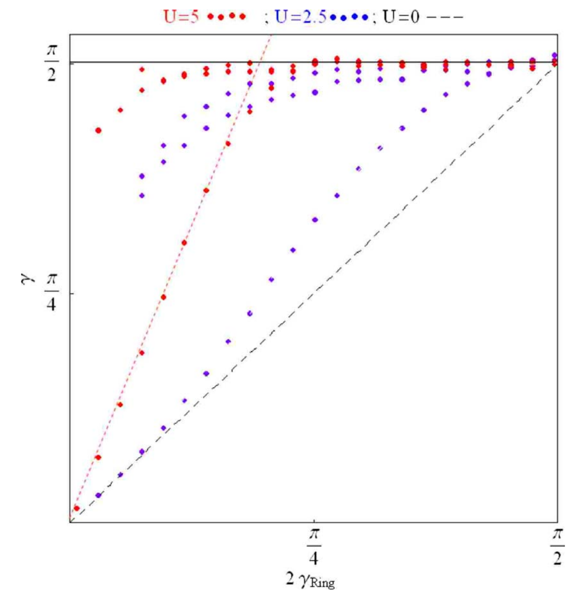

FIG. 5. (Color online) The position of the peaks in the plane $\gamma_{\rm ring}-\gamma$. At one fixed value of $\gamma_{\rm ring}$ and potential U, the spin polarized angle γ can have several values corresponding to the different values of resonant peaks found at different k (see, e.g., on top of Fig. 4, where different values of resonant peaks are found at different k, for a fixed value of the SOI strength). If the barriers vanish, the rotation angle corresponds to the one obtained by the AC phase. When the strength of barriers increases, the rotation angles corresponding to the peaks are mainly located near the fixed point $\gamma=\pi/2$. However, for small values of $\gamma_{\rm ring}$ and strong values of U, some peaks in the transmission are located along a straight line, according to the heuristic relation $\gamma\sim\eta(2\gamma_{\rm ring})$, where η depends on the construction parameters (d,R,U) of the device. The slope η increases with U. Thus the angles γ and $2\gamma_{\rm ring}$ can coincide just for the *fixed points* $\gamma=0+n\pi$. However, as we show on the top right of Fig. 4, different values of resonant peaks are found at different k, for a fixed value of the SOI strength.

VIII. QUANTUM RING AS A LOGIC GATE

In the language of quantum informatics,[15] the transformation discussed above represents a rather general single-qubit gate and it shows that in principle a continuous set of spin rotations can be achieved already with a single diametrically connected ring. A transformation with $\gamma=\pi/2$ is essentially a so-called Hadamard gate,[15] which plays a distinguished role in quantum algorithms, while two such gates in series result in a X gate or quantum NOT (inverter) gate.

In the case of arbitrary γ, other types of transformations can be realized. We note that in principle a number of other gates can be constructed by coupling several rings, as discussed in Ref. 16. This should be realized with parameters corresponding to unitary gates, so that the product of the corresponding spin rotations results again in a lossless current transformation. This is quite difficult in the single electron-tunneling regime, where the transmission peaks are often significantly lower than one.

As emphasized in some papers in the past,[20] similar rings in the presence of an external magnetic field can be used for spin filtering. This points to the possibility to integrate gates and filters that can serve as elementary building blocks of a quantum network based on spin sensitive devices.[28]

The present calculation was done for an idealized model system in which transport is strictly one dimensional, i.e., the finite width of the ring wire was not included. Our narrow ring and wires imply the assumption of single-mode propagation. Next we discuss some realistic or theoretical devices capable of acting as spintronic gates based on the RSOI.

A realistic ring has radius $R\sim 0.2-0.4\ \mu$m and, according to Ref. 16, in InGaAs and for a Fermi energy of order 10 meV, the value of $2\gamma_{\rm ring}$ can be modulated between 0 and $0.8\pi/2$. However the value of the RSOI could also be tuned by controlling an external transverse electric field. A typical device should be made by using wires and a ring of width W between 50 and 150 nm, corresponding to an effective width $l_\omega\sim 10-30$ nm. The distance d between the ring-lead junctions has to be much larger than l_ω and it can be assumed between 0.2 and 0.6 μm. The small width of the ring and wires justifies the assumption of a 1D model because the excitation gap due to the transverse motion can be assumed as $\Delta\sim 50$ meV, which is quite larger than the energy scale associated to the kinetic motion $[\varepsilon \sim \hbar^2 k^2/(2m*)]$.

In summary, we studied the crossover between the ballistic transport regime and the single electron-tunneling regime through a ring in the presence of Rashba SOI of strength α. We showed that by modulating the distance between the junctions and the external electric field, it is possible to obtain a logic gate which could have potential applications in quantum computing. This device similar to the one proposed in Ref. 16 works on a single electron qubit and could be easily applied in quantum spintronics.

Thus the proposed device can lead to different effects in the spin state transformation of electrons assumed as flying spin qubits. We point out that our device, as the ones reported in Ref. 29, acts on the single electron spin and can be assumed as a logic gate operating on a single logic variable. However recently some researches were oriented to engineer spintronics nanodevices operating with more than one electron. Thus, in the last years, some mesoscopic systems were proposed to obtain entanglement between static and flying qubits;[30] another important task in the development of quantum computing in the solid state. In fact it has recently been shown theoretically that the spin-dependent scattering of a propagating electron from a bound electron is sufficient to give full entanglement between the qubits embodied in their respective spin.[31] Thus in the recent literature[30,32] a generalized real-space Anderson model is introduced for a quasi-one-dimensional structure consisting of a binding site (the static qubit) coupled to ideal leads. A future task in the development of quantum computing in the solid state will be the engineering of these two different kinds of devices in series.

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